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P. VAN EMDE BOAS & P.M.B. VITÁNYI

A NOTE ON THE RECURSIVE ENUMERABILITY OF SOME CLASSES OF RECURSIVELY ENUMERABLE LANGUAGES

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A note on the recursive enumerability of some classes of recursively enumerable languages

by

P. van Emde Boas & P.M.B. Vitányi

ABSTRACT. An elementary proof is presented for the fact that the class of infinite recursive languages is not recursively enumerable. Relevance for contemporary linguistics and computer science is explained.

Both in linguistics and in the theory of programming languages, one is interested in those languages in which membership is decidable: the *recursive* languages. For each such language there is a procedure which allows one to decide effectively whether or not a given sentence belongs to the language. Even so, it can be the case that, given a "grammar" which generates each sentence of the language, we do not have the appropriate procedure available, or, if we have, the procedure is not feasible, i.e., it does not give answers to reasonable questions within a reasonable time. The theoretical machinery introduced to generate classes of languages, like transformational grammars in linguistics [1] or van Wijngaarden grammars in the theory of programming languages [8], consists of giving for each language in the class an effective procedure ("grammar") which, during some non-terminating computation, produces each sentence belonging to the language. Given a sentence and such a procedure we can be in the position that, at each stage of the computation, we only know that the sentence has not occurred yet but we have no way of knowing whether it is ever going to appear. The most general class of languages generable by such effective means is the class of *recursively enumerable* languages and this is precisely what we obtain by the usual powerful mechanisms such as transformational grammars and van Wijngaarden grammars, [5] and [8]. Therefore, people have looked for restrictions such that the resulting class of languages contains all and only recursive languages. Furthermore, we want a method to provide uniformly, for each "grammar" satisfying the restriction, a decision procedure for checking membership in the language generated by that "grammar". Previous proposals for restrictions on transformational grammars either do not reduce the generating power [4], reduce it too strongly, excluding potentially interesting recursive languages [6], or leave it undecidable whether a given grammar satisfies the restriction, i.e. the restriction itself is not recursive [5]. These failures are due to the fact that although we can recursively enumerate a class of effective prodecures generating exactly the recursive languages, there exists no recursive enumeration of corresponding decision procedures for membership in these languages, cf. DEKKER [2].

Stimulated by JANSSEN, KOK & MEERTENS [3] who investigated these matters and reformulated and sharpened one of DEKKER's results, we found that the class of infinite recursive languages is not recursively enumerable. Although this fact presumably belongs to the folklore of recursion theory we present in this note an elementary proof, so as to proliferate these facts among interested linguists etc. to whom the original references are less accessible. The class seems relevant since in practice people are not concerned with grammars generating finite languages and might wish to get rid of them.

For definitions and terminology from recursion theory we refer to ROGERS [7,chs.1-5]. Let $\phi_1,\phi_2,\ldots$ be an effective enumeration of partial recursive functions. A *class* C of recursively enumerable sets is *recursively enumberable* if there exists a recursively enumerable set A such that $C = \{\text{range } \phi_i \mid i \in A\}$.

To derive the desired result we prove the slightly stronger:

<u>THEOREM</u>. *There exists no recursively enumerable class of infinite recursively enumerable sets which contains all infinite recursive sets.*

<u>PROOF</u>. Assume the contrary and let $C = \{\text{range } \phi_i \mid i \in A\}$ be such a class. Since clearly A is infinite we may assume that A = range σ for some total recursive function σ. Let $\psi_i = \phi_{\sigma(i)}$ for all i. Now $C = \{\text{range } \psi_i \mid i \in \mathbb{N}\}$. We describe a procedure for effectively enumerating two disjoint sets X and Y. At stage 1 we simulate ψ_i until the first two distinct elements x_1 and y_1 of the range of ψ_i have been computed. Put x_1 in X and y_1 in Y. At stage $k > 1$ we simulate ψ_k until the first two distinct elements x_k and y_k of the range of ψ_k have been computed which are both greater than all elements in $\{x_1,\ldots,x_{k-1}\} \cup \{y_1,\ldots,y_{k-1}\}$. Put x_k in X and y_k in Y. Since by assumption range ψ_i is infinite for all i all stages terminate and consequently the effectively enumerated sets $X = \{x_1,x_2,\ldots\}$ and $Y = \{y_1,y_2,\ldots\}$ are infinite. By construction, $X \cap Y = \emptyset$. Since X is infinite and effectively enumerated

in strictly increasing order X is an infinite recursive set and there is an index j such that X = range ψ_j. But $y_j \in$ range $\psi_j \cap Y \subseteq X \cap Y = \emptyset$: contradiction. □

COROLLARY. *The class of infinite recursive languages is not recursively enumerable.*

The corollary should be compared with the better known results of DEKKER stating (i) the class of recursive languages is recursively enumerable, (ii) the class of infinite recursively enumerable languages is not recursively enumerable.

REFERENCES.

1 CHOMSKY, N., *Aspects of the Theory of Syntax,* MIT Press, Cambridge, Massachusetts (1965).

2 DEKKER, J.C.E., *The Constructivity of Maximal Dual Ideals in Certain Boolean Algebras,* Pacific J. Math. 3 (1953) 73-101.

3 JANSSEN, Th.M.V., KOK, G. & MEERTENS, L.G.L.Th., *On Restrictions on Transformational Grammars Reducing the Generative Power,* Mathematical Center Tech. Rept. ZW 41/75, M.C., Amsterdam.

4 PETERS Jr., P.S. & RITCHIE, R.W., *On Restricting the Base Component of Transformational Grammars,* Inf. Contr. 18 (1971) 483-501.

5 PETERS Jr., P.S. & RITCHIE, R.W., *On the Generative Power of Transformational Grammars,* Inf. Sciences 6 (1973) 49-83.

6 PUTNAM, H., *Some Issues in the Theory of Grammar,* In: Proc. Symp. Appl.

Math. vol. XII: The Structure of Language and its Mathematical Aspects (R. Jacobson ed.), Am.Math.Soc., Providence, N.J. (1961).

7 ROGERS, H., *The Theory of Recursive Functions and Effective Computability*, McGraw Hill, New York (1967).

8 SINTZOFF, M., *Existence of a van Wijngaarden Syntax for Every Recursively Enumerable Set*, Ann. Soc. Scientifique de Bruxelles T. 81, II (1967), 115-118.